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Landau level bosonization of a two-dimensional electron gas

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In this work we introduce a bosonization scheme for the low-energy excitations of a two-dimensional interacting electron gas in the presence of a uniform magnetic field under conditions where a large integral number of Landau levels are filled. We give an explicit construction for the electron operator in terms of the bosons. We show that the elementary neutral excitations, known as the *magnetic excitons* or *magnetoplasma modes,* can be described within a bosonic language and that it provides a quadratic bosonic Hamiltonian for the interacting electron system that can be easily diagonalized. [S0163-1829(97)51412-3]

Bosonization of Fermi fields has been an important nonperturbative tool for finding the solutions to problems involving many interacting fermions. The pioneering attempts by Bloch[1] on sound waves in dense one-dimensional (1D) systems, which were later extended and given an intuitive foundation by Tomonaga[2], called attention to the conceptual and operational simplicity that results if all excitations of a fermionic system can be described within a bosonic language.[3–5] Nowadays, the 1D bosonization is a mature field.[6] Bosonization in higher dimensions was pioneered by Luther[7] and extended by Haldane a few years ago.[8] More recently this approach has been applied in the context of strongly correlated systems where the possibility of nonconventional ground states seeks for a nonperturbative method of study.[9–11]

In the context of correlated systems, one of the most interesting phenomenon in condensed matter physics is the quantum Hall effect[12–14] either in its integer (IQHE) or fractional (FQHE) regimes. Despite the fact that there is no thermodynamic distinction between these two phenomena,[13] they have quite different microscopic origins. The IQHE can be understood from the properties of noninteracting electrons whereas the understanding of the FQHE necessarily requires a treatment of the electron-electron interaction. Consequently, almost all the efforts to study correlations in a 2D electron gas in the presence of a magnetic field B were directed toward the study of the strong magnetic field case where the FQHE takes place. In this case the Landau level spacing, the cyclotron energy $\hbar\omega_c=\hbar eB/(mc)$, can be much greater than the typical Coulomb energy, which is of the order of e^2/ℓ [where $\ell=\sqrt{c\hbar/(eB)}$ is the magnetic length], and therefore the system can be described only in terms of the first Landau level and the mixing with the other Landau levels can be neglected or taken into account perturbatively. In a weak magnetic field[15] this is not the case and the typical Coulomb energy can exceed the cyclotron energy resulting in a mixing between the Landau levels. Some attempts were made to develop a method for studying the mixing caused by these correlation effects starting from a Fermi liquid theory[16] or Green's-function method.[15,17–19]

In this paper we start from a Landau level description of the system and introduce a nonperturbative bosonization scheme for the 2D interacting fermion gas subject to a perpendicular uniform magnetic field under the condition that there is a large integer number of filled Landau levels. In this case, the excitations are electron-hole pairs often called magnetic excitons and as we will show can be described within a bosonic language.

Recent experiments in tunneling between parallel 2D electron gases in applied magnetic fields[20] have revealed the existence of a gap in the tunneling density of states. In this case it is established that correlations have a major hole in the tunneling conductance and we expect that the method presented here can be applied to such problems where the correlations between the electrons in a magnetic field are so important.

We start with the Hamiltonian of a spinless fermion gas in a plane perpendicular to a uniform magnetic field

$$H_o=\frac{1}{2m}\int d^2r\Psi^\dagger(\mathbf{r})\left(\mathbf{p}-\frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2\Psi(\mathbf{r}),$$

where $\Psi(\mathbf{r})$ is the fermion field operator (for the time being we do not discuss the spin of the electron).

Rewriting $\Psi(\mathbf{r})$ in a Landau level basis (symmetric gauge) the Hamiltonian H_0 becomes diagonal,

$$H_o=\hbar\omega_c\sum_{n,m=0}^{\infty}(n+1/2)c^\dagger_{n,m}c_{n,m},$$

where $c^\dagger_{n,m}$ is the fermion operator that creates a particle in a Landau level n with the guiding center at a distance $\sqrt{2m+1}\ell$ from the origin of the coordinate system. Therefore the m degeneracy is related to the location of the position of the guiding center of the cyclotron orbit. The noninteracting ground state is generated by uniformly filling ν Landau levels (ν integer) of each guiding center labeled by the quantum number m

$$|G\rangle=\prod_{m=0}^{N_\phi}\prod_{n=0}^{\nu-1}c^\dagger_{n,m}|0\rangle,$$

where $N_\phi=(S/2\pi\ell^2)$ is the number of flux quanta. The neutral excitations in the noninteracting system can be generated by creating a hole in a Landau level $p\leqslant\nu-1$ at the guiding center m and an electron in a level $n+p\geqslant\nu-1$ at the guiding center m'.

We note, however, that it is not possible to define eigenstates of the guiding center $\vec{R}_{0i}$ for the cyclotronic motion of a single charged particle because of the noncommutativity of the components of the position operators. Nevertheless we can define a vector $\vec{R}_0=\vec{R}_{0e}+\vec{R}_{0h}$, related to the center of mass of the guiding centers of the electron and the hole excited in the system, whose components x_0 and y_0 commutes with each other and therefore have simultaneous eigenstates. It is also possible to define a momentum operator $\vec{P}_0$, canonically conjugated to $\vec{R}_0$, which is related to the momentum of the center of mass of the guiding centers. The eigenstates of this operator can be generated by a superposition of the above described neutral excitations in different guiding centers and they form a complete basis in the space of the neutral excitations known as *magnetic excitons.*[18] In operator form they can be described as

$$e^\dagger_{n,p}(\mathbf{k})|G\rangle=e^{-(k\ell)^2/4}\sum_{m',m=0}^{\infty}G_{m',m}(\ell k)c^\dagger_{n+p,m'}c_{p,m}|G\rangle,$$

where $k=k_x+ik_y$, $\kappa=|k|$, and $G_{m',m}$ is given by[14]

$$G_{n+p,p}(k)=\sqrt{\frac{p!}{(n+p)!}}\left(\frac{-ik}{\sqrt{2}}\right)^n L_p^n\left(\frac{\kappa^2}{2}\right)$$

with L_p^n being generalized Laguerre polynomials.

The density fluctuations of the system can be expressed in terms of superpositions $a^\dagger_n(\mathbf{k})$ of these magnetic excitons

$$\delta\rho(\mathbf{k})=\sum_{n=1}^{\infty}\{a^\dagger_n(\mathbf{k})+a_n(-\mathbf{k})\},$$

where

$$a^\dagger_n(\mathbf{k})=e^{-(\kappa\ell)^2/4}\sum_{p=0}^{\infty}G_{n+p,p}(\ell k^*)e^\dagger_{n,p}(\mathbf{k}) \qquad (1)$$

can be interpreted as *magnetic plasmons* and $\rho_0=(\nu/2\pi\ell^2)$ is the electronic average density in the system.

In order to bosonize this problem we proceed exactly as in the Luttinger model and enlarge the Hilbert space of the fermionic system by including filled energy eigenstates of negative values of the quantum number n. This construction is the analogue of the sea of infinite negative energy of the Luttinger model (the Tomonaga construction can also be obtained in an analogous way by making the Hilbert space finite[21]). This vacuum does not concern us because we are studying the *low-energy excitation* spectrum of the system. It is clear from this construction that the bosons only make a faithful representation of the electronic system if $\nu\gg1$. In the same way the bosonization of the electron gas is only possible when the system is very dense (large Fermi momentum). By doing that one can show that the bosonic operators defined in Eq. (1) obey the *exact* commutation relations,

$$[a_n(\mathbf{k}),a^\dagger_{n'}(\mathbf{k}')]=\delta_{n,n'}\delta_{\mathbf{k},\mathbf{k}'}N_\phi\mathcal{G}_n(k\ell),$$

$$[a_n(\mathbf{k}),a_{n'}(\mathbf{k}')]=0,$$

where

$$\mathcal{G}_n(kl)=nJ_n^2(kR_c),$$

J_n is a Bessel function of the first kind, and $R_c=\sqrt{2\nu}\ell$ is the size radius of the last occupied ''orbit.'' We can proceed further and define the canonical bosonic operators

$$b_n(\mathbf{k})=\frac{1}{\sqrt{N_\phi\mathcal{G}_n(k\ell)}}a_n(\mathbf{k})$$

and recover the standard bosonic commutation relations.

It is easy to show that in terms of these bosonic degrees of freedom the free Hamiltonian can be rewritten, apart from a background energy, as

$$H_o=\hbar\omega_c\sum_{\mathbf{k}}\sum_{n=1}^{\infty}nb^\dagger_n(\mathbf{k})b_n(\mathbf{k}), \qquad (2)$$

which describes the energy of an assembly of noninteracting bosons.

So far we have succeeded in expressing the neutral excitations of the system of electrons in terms of bosons. In order to complete our scheme one needs to express the electron operator in terms of bosons. This is done by transforming from the momentum representation, which is the natural representation for bosons, to the $|nm\rangle$ representation, which is the appropriate one for fermions in a magnetic field. This should be contrasted with the case of the electron gas where the momentum is a good quantum number for the electron and the particle-hole excitation. It means that we must construct an operator that creates an electron in a Landau level n and guiding center m as a function of the density fluctuations within this guiding center. This transformation involves a linear superposition of the magnetic excitons at different momentum states. This is done by considering the operator

$$a^\dagger_{n,m}=\frac{1}{\sqrt{n}}\sum_{p=-\infty}^{\infty}c^\dagger_{n+p,m}c_{p,m}$$

$$=2\pi\sqrt{N_\phi}\int d^2\mathbf{k}\,e^{-\kappa^2/4}G_{m,m}(k^*)b^\dagger_n(\mathbf{k}).$$

Different from the operator $b^\dagger_n(\mathbf{k})$ that creates density fluctuations with a definite momentum $\mathbf{k}$, the operator $a^\dagger_{n,m}$ generates density fluctuations on a definite guiding center. This operator also obeys exact bosonic commutation relations. This is completely analogous to Luther's[7,8] construction of the Fermi liquid in 2D and 3D as many 1D fermi liquids, one for each direction normal to the Fermi surface. But contrary to that construction here we do not need to ''smear'' the operators close to the Fermi surface. The discrete nature of the Landau states does this smearing naturally. Here we have also reduced the 2D problem of electrons in a magnetic field to N_ϕ 1D *chiral* problems, one for each guiding center labeled by m.

In order to complete the bosonic description of the fermion operator it will be necessary to define an operator that creates a fermion in a ''phase'' state associated with this circular motion. This is done by rewriting the original fermion operator as

$$c_m(\theta)=\frac{1}{\sqrt{2\pi}}\sum_{n=-\infty}^{\infty} e^{-in\theta}c_{n,m},$$

where we have used the enlarged Hilbert space and it is easy to show that

$$\{c^\dagger_{m'}(\theta'),c_m(\theta)\}=\delta_{m',m}\delta(\theta'-\theta), \tag{3}$$

$$\{c_{m'}(\theta'),c_m(\theta)\}=0. \tag{4}$$

The operator $c^\dagger_m(\theta)$ creates an electron about the guiding center given by the quantum number m and at the phase θ of the cyclotronic motion. This is the operator that can be properly bosonized. Indeed the Mandelstam form[4,5] of the fermion operator is given by

$$c^\dagger_m(\theta)=\frac{1}{\sqrt{2\pi}}e^{+i\nu\theta}e^{i\phi^\dagger_m(\theta)}U_m e^{i\phi_m(\theta)},$$

with

$$\phi_m(\theta)=\frac{N_m\theta}{2}+i\lim_{\alpha\to 0}\sum_{n=1}^{\infty} e^{-n\alpha/2}\frac{e^{-in\theta}}{\sqrt{n}}a_{n,m}$$

and

$$U_m=e^{i\pi\sum_{s=0}^{m-1}N_s}e^{i\Theta_m},$$

where the Hermitian operators

$$N_m=\sum_{p=-\infty}^{\infty}(c^\dagger_{p,m}c_{p,m}-\langle c^\dagger_{p,m}c_{p,m}\rangle)$$

and Θ_m are canonically conjugated ($[N_m,\Theta_{m'}]=i\delta_{m'm}$) such that the unitary operators U_m anticommute.

In this representation, it can be shown that the fermionic operators obey the desired anticommutation relations (3) and (4) and it recovers the long time asymptotic behavior of the noninteracting propagator between two different phase states that is given by

$$K(\theta,t)=\langle c^\dagger_{m'}(\theta,t)c_m(0,0)\rangle$$
$$=\frac{\delta_{m',m}}{2\pi}\frac{e^{+i\nu\theta}}{1-e^{+i(\theta-\omega_c t)}}$$

and can be related to the space-time propagator by writing the operator $\Psi(\mathbf{r})$ in terms of the $c_m(\theta)$.[21] That completes the construction of the model. Now we have an *operator* relation between electrons in a magnetic field and bosons.

Our next step is to include the interaction between the electrons within this framework. In a uniform neutralizing background the total Hamiltonian can be written as

$$H=H_o+H_I,$$

where H_o is given by Eq. (2) and

$$H_I=\frac{1}{2\pi\ell^2}\sum_{\mathbf{k}} V(k)\sum_{n,n'=1}^{\infty}\sqrt{\mathcal{G}_n(k\ell)\mathcal{G}_{n'}(k\ell)}$$
$$\times[b^\dagger_n(\mathbf{k})+b_n(-\mathbf{k})][b^\dagger_{n'}(-\mathbf{k})+b_{n'}(\mathbf{k})]$$

is a density-density interaction between the fermions, which is written in bosonic language. $V(k)$ is the Fourier transform of the interacting potential.

In order to diagonalize H we will adopt a generalized Bogoliubov transformation[10] with coefficients $\mu_{nl}(\mathbf{k})$ and $\vartheta_{nl}(\mathbf{k})$ that is given by

$$b_n(\mathbf{k})=\sum_{l=1}^{\infty}[\mu_{nl}(\mathbf{k})\beta_l(\mathbf{k})+\vartheta_{nl}(\mathbf{k})\beta^\dagger_l(-\mathbf{k})],$$

where $\beta_l(\mathbf{k})$ are the new bosonic operators. This canonical transformation yields

$$H=\hbar\omega_c\sum_{\mathbf{k}}\sum_{l=1}^{\infty}\Omega_l(\mathbf{k})\beta^\dagger_l(\mathbf{k})\beta_l(\mathbf{k}),$$

where the new eigenfrequencies $\Omega_l(\mathbf{k})$ are determined by the equation

$$1=\left(\frac{V(k)}{2\pi\hbar\omega_c\ell^2}\right)\sum_{n=1}\frac{\mathcal{G}_n(k\ell)n}{[\Omega_l(\mathbf{k})^2-n^2]}.$$

The above equation has the same form as the one obtained for the normal modes in the integer quantum Hall effect in the random-phase approximation for a Coulomb potential $V(\kappa)=2\pi e^2/\kappa$.[18] Moreover, the same equation is obtained in the fractional quantum Hall effect for composite fermions.[19] Now we have shown that as far as the bosonization construction is valid, the equation that defines the eigenvalues holds for any value of the magnetic field. This is a consequence of the fact that bosonization fulfills the Ward identities. Thus exactly as in the Fermi gas, the limit of validity of bosonization goes beyond what is expected.

As a final comment we would like to mention that the method developed in this paper can be applied to the fractional Hall effect if we use the Chern-Simons theory to map the original electrons into the composite fermions. In this case, as is well established, the fractional quantum Hall effect is mapped into the integer quantum Hall effect of composite fermions. Thus the condition $\nu\gg 1$ becomes equivalent to looking at fillings close to 1/2.[22] The nature of the ground state of this system has recently been the source of much controversy and we believe that our methods can provide nonperturbative answers for the structure of the ground state of this problem.[21]

In conclusion, we have proposed a method of bosonization for 2D fermions subject to a uniform magnetic field by directly working in a Landau basis. We propose an analogue of the Luttinger model that can be bosonized and provides an operator relation between electrons and bosons in the limit of $\nu \gg 1$. This construction is essentially nonperturbative and provides new insights into the problem of electrons interacting in the presence of magnetic fields. We show that bosons are collective excitations of the system that are related with a collection of N_ϕ 1D Luttinger liquids associated with each guiding center. We also obtain the dispersion of the collective modes of the system and discuss the applicability of the method to the integer and fractional quantum Hall effects. We expect our method will enlarge the understanding of the physics of strongly correlated systems in the presence of magnetic fields.

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